Homework 8 – 4.2, 4.3

# Section 4.2

## Problem 5.

The following data give approximations to the integral

Assuming , construct an extrapolation table to determine .

From the Lecture notes:

Now to solve for :

First :

Now for :

Now :

Thus:

Note that:

## Problem 8.

The forward-difference formula can be expressed as

Use extrapolation to derive an for .

Now consider:

This gave us an extrapolation of order . Now for :

# Section 4.3

## Problem 1.

Approximate the following integral using the Trapezoidal rule.

The Trapezoidal rule the following:

Where , and .

Using a program:

#include <iostream>

using namespace std;

//Will contain the function.

double function(double);

//Will perform the Trapezoid method.

double Trapezoid(double, double);

int main()

{

cout << Trapezoid(1, 1.6) << endl;

}

double function(double x)

{

double f = 0;

//Function.

f = (2 \* x) / (pow(x, 2) - 4);

//Returns value.

return f;

}

double Trapezoid(double x0, double x1)

{

double integral = 0;

double h = (x1 - x0);

//Trapezoid method.

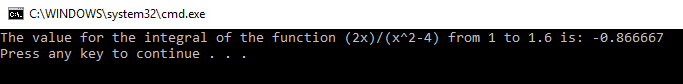
integral = (h / 2) \* (function(x0) + function(x1));

//Returns the integral.

return integral;

}

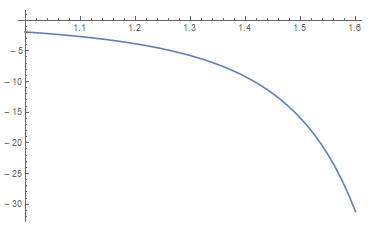
We get that:



## Problem 3.

Find a bound for the error in Exercise 1 using the error formula, and compare this to the actual error.

Plotting the graph for the second derivative:



We get that the greatest value in magnitude occurs at .

From here we can get that the error bound is:

Using Mathematica to compute the integral:

The absolute and relative errors are:

## Problem 5.

Repeat Exercise 1 using Simpson’s rule.

Simpson’s Rule is:

Note that , and .

Using a program:

#include <iostream>

using namespace std;

//Will contain the function.

double function(double);

//Will perform the Trapezoid method.

double Simpson(double, double);

int main()

{

double a = 1, b = 1.6;

cout << "The value for the integral of the function (2x)/(x^2-4) from " << a << " to " << b << " is: ";

cout << Simpson(a, b) << endl;

}

double function(double x)

{

double f = 0;

//Function.

f = (2 \* x) / (pow(x, 2) - 4);

//Returns value.

return f;

}

double Simpson(double x0, double x2)

{

double integral = 0;

double h = (x2 - x0) / 2;

double x1 = x0 + h;

//Simpson method.

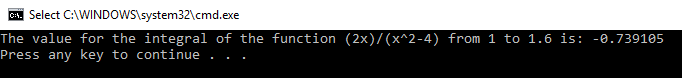
integral = (h / 3) \* (function(x0) + 4 \* function(x1) + function(x2));

//Returns the integral.

return integral;

}

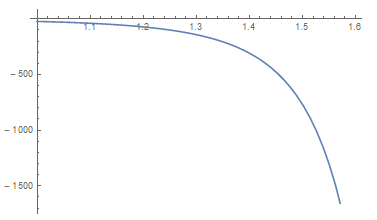
We get that:



## Problem 7.

Repeat Exercise 3 using Simpson’s Rule.

Plotting the graph for the fourth derivative:



We get that the greatest value in magnitude occurs at .

The absolute and relative errors are: